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Designing canopy tubes for tunnel stability in weak ground: A comparison of analytical and 3D numerical approaches

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ABSTRACT: The use of canopy tubes as pre-support in weak ground is a widely adopted method to control deformation in tunnelling. By redistributing ground loads radially and longitudinally, canopy tubes enhance tunnel stability and reduce the risk of collapse. Many analytical and semi-empirical methods have been proposed. Three-dimensional numerical modelling typically remains impractical for routine design due to computational demands. As a result, designs often rely on simplified approaches. This paper reviews available methods, focusing on two representative analytical models: a simplified beam-on-elastic-foundation method by John & Mattle (2002) and a more refined Pasternak foundation model by Wang et al. (2009). These are benchmarked against finite element modelling results for underground conditions representative of Sydney sandstone and shale. The study assesses the analytical methods in conjunction with equivalent finite element models, highlighting their capabilities and limitations, and explores the implications for practical canopy tube design.

1 INTRODUCTION

Canopy tubes (pipe umbrellas) are a well-established pre-support method for tunnelling in weak ground. By redistributing loads radially and longitudinally, they improve roof stability and reduce the risk of deformation or collapse, supporting ground between the installed lining and the tunnel face.

Although their benefits are clear, the load transfer mechanisms are complex. Full 3-Dimensional (3D) numerical modelling can capture these mechanisms in detail but is rarely practical for routine design due to its computational cost. As a result, design typically relies on simplified analytical or semi-empirical methods, which provide conservative estimates based on assumed boundary conditions and input parameters.

This paper reviews selected analytical and semi-empirical methods for canopy tube design, focusing on two: the simplified beam model of John & Mattle (2002) and the beam-on-elastic-foundation model of Wang & Jia (2008) and Wang et al. (2009). Their predictions are benchmarked against 3D finite element modelling for Sydney sandstone and shale conditions, highlighting the influence of modelling assumptions, boundary conditions, and the capabilities and limitations of each approach.

2 LITERATURE REVIEW

Various analytical and semi-empirical methods have been developed to analyse canopy tube design actions in weak ground conditions. Oke et al. (2016) provides an excellent overview of the various semi-analytical methods that are available. Oke et al. (2016) broadly classifies the semi-analytical solutions available into two categories: simplified beam models (Harazaki et al., 1998;

John and Mattle, 2002) and elastic foundation models (Wang & Jia, 2008; Yuan et al., 2013; Song et al., 2013).

2.1 *Simplified beam models*

Simplified beam models idealise the canopy tubes as isolated beams subjected to external loads, neglecting the interaction between the ground and the structure. Harazaki et al. (1998), as cited by Oke et al. (2016), employed such a model based on field observations from the Maiko Tunnel in Kobe, Japan. Their analysis, which involved a simplified beam model to capture the moment profile of the canopy tube, interpreted the observed bending moment behaviour of the steel pipes in the field data as being similar to that of a beam simply supported on two pivots and subjected to a distributed load. John & Mattle (2002) used a similar approach, with relatively simple boundary conditions that incorporate the ground support without explicitly modelling soil stiffness, which is elaborated further in Section 2.3.1.

2.2 *Elastic foundation models*

Elastic foundation models conceptualise canopy tubes as beams resting on a continuous elastic medium, incorporating ground parameters to estimate spring stiffness and thereby more realistically represent ground-structure interaction. Song et al. (2013) developed a simplified theoretical beam-spring structural analysis model based on elastic foundation theory to calculate bending moment and shear forces. Their findings show that the factor of safety (FOS) against bending is generally lower than the FOS against shear, suggesting that pipes are more susceptible to failure in bending than in shear. Similarly, Yuan et al. (2013) modelled the pipe roof as a beam on an elastic foundation to examine how excavation-induced loads are redistributed by the pipe roof to the supporting structure and the unexcavated soil ahead of the tunnel face. Wang & Jia (2008), Wang et al. (2009), proposed an enhancement to the Winkler Spring model by adopting the Pasternak elastic foundation model, which incorporates the shear interaction between adjacent springs and can potentially yield improved predictions of ground deformation. Although analytically more complex, these models offer a more realistic representation of the load transfer mechanism in front of the tunnel face.

2.3 *Analytical methods for canopy tube design*

Among the various available methods, this paper focuses on two representative analytical approaches: the simplified beam model described by John & Mattle (2002) and the Pasternak elastic foundation model developed by Wang & Jia (2008), Wang et al. (2009). Sections 2.3.1 and 2.3.2 review these methods in detail and are subsequently evaluated against 3D numerical modelling results to assess their practical relevance and applicability in tunnel support design.

2.3.1 *John & Mattle (2002)*

This paper presents a simplified structural analysis method for the analysis of canopy tubes, modelling them as longitudinal beams supported at one end by the tunnel lining and embedded at the other end in the ground ahead of the tunnel face. A series of numerical analyses were conducted using the Finite Element Method (FEM) with an axisymmetric model. The results indicated that the bending moment distribution resembled that of a beam on elastic support, showing smaller negative and positive values near the supported end and larger values closer to the unsupported region. This was idealised in a simplified structural model as an equivalent beam hinged at the tunnel lining and fixed at the face, as shown in Figure 1a. The effective length of this equivalent beam was taken as 1.5 times the unsupported length based on consideration of a number of case studies. Bending moments calculated using this simplified model showed good agreement with the bending moments from the FEM analysis. Additionally, the study found that the ground loads estimated through FEM were in good agreement with those calculated using Terzaghi's silo theory.

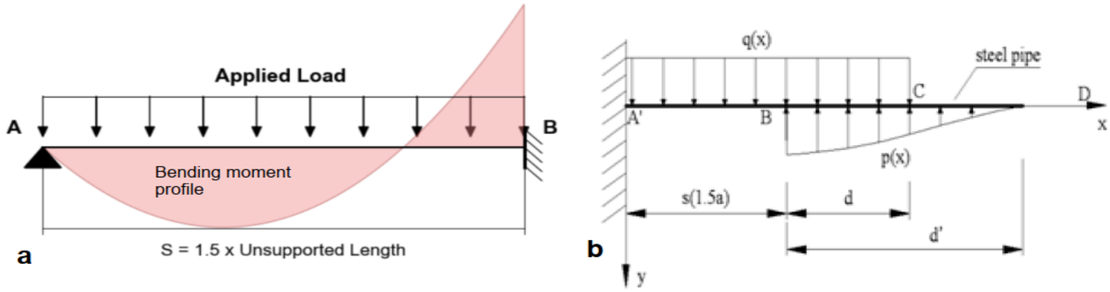

Figure 1. a). Analytical model proposed by John & Mattle (2002) b). Mechanical model of pipe roof reinforcement during tunnel excavation (Wang & Jia. 2008)

2.3.2 *Wang & Jia (2008)*

Wang and Jia (2008) developed an analytical model to assess pipe roof reinforcement ahead of tunnel faces. The model uses the Pasternak elastic foundation beam theory, which includes shear interaction, an improvement over the Winkler model. The model assumes a semi-infinite beam on a Pasternak elastic foundation, fixed at one end to represent the tunnel lining. The fixed end has prescribed vertical displacement (to simulate staged excavation) and no rotation. The free end, positioned ahead of the tunnel face and beyond the failure zone, is assumed to have negligible movement or rotation.

Geometry and Load Definition (see Figure 1b):

- Pipe is fixed at Point A′, located 0.5 longitudinal spans behind the leading edge of the initial shotcrete support.
- Pipe extends to Point B, with a total span of:
  - s=1.5× unsupported excavation length
- The active failure zone ahead of tunnel face is assumed to extend for a length, d = h·tan(45°−ϕ/2), where ϕ is soil friction angle. Overburden pressure q(x) is assumed to act up to this distance, indicated as C in Figure 1b.
- Length of pipe ahead of the face, d′ will be greater than d.

The pipe carries downward overburden pressure *q(x)* and upward subgrade reaction *p(x)* from the ground ahead of the face. Wang et al. (2008) combined beam theory with a Pasternak elastic foundation to derive a fourth-order equation for vertical deformation ω(x), from which bending moments are calculated. Both Wang & Jia (2008) and John & Mattle (2002) estimate loads using Terzaghi's silo theory but differ in boundary conditions: Wang & Jia assume full fixity at the lining, while John & Mattle place it at the unexcavated face.

In practice, abutment conditions vary, so both models use conservative simplifications:

- Simplified beam: Rigid face support, hinged support 0.5 spans behind the lining; tube spans 1.5× unsupported length.
- Elastic foundation: Elastic support beyond the face, fixed support 0.5 spans behind the lining; fixed in rotation but allowed vertical deflection.

Both assume one (elastic) or two (simplified) rigid/near-rigid abutments, acknowledging that face and lining support are rarely fully effective, consistent with case histories.

3 COMPARATIVE ASSESSMENT

This section compares the two analytical approaches by comparing their predictions against results from three-dimensional finite element analysis (FEA) using PLAXIS 3D version 2024.2 software. The analysis methods used are presented in Table 1. Bending moments were extracted from each method to evaluate canopy tube behaviour under varying tunnel and ground conditions.

A series of tunnel configurations were analysed, varying by:

- Ground conditions: Sydney Sandstone Class V and Shale Class V
- Tunnel Heading heights: 7 m and 8.5 m
- Overburden depths: 5 m, 10 m, 15 m and 20m.

Table 1. Analysis methods used for comparative assessment

Comparative Assessment performed		Objective
Section 4.1 Comparison of Analytical models	Simplified Beam Model vs. Beam on Elastic Foundation	Compare analytical methods and plot moment distribution profiles under identical conditions using full overburden load. Assess the impact of boundary conditions and ground stiffness ahead of tunnel on bending moment predictions.
	Beam on Elastic Foundation: Impact of Heading height	Assess the influence of tunnel heading height on bending moment predictions using beam on elastic foundation.
	Ground Load using Terzaghi's Silo theory for various overburdens	Quantify the sensitivity of maximum bending moment to overburden depth using Terzhaghi's silo loads. Compare the moment profiles with full overburden loads
Section 4.2 Idealised FEA models	Beam on Elastic Foundation vs. FEA Validation Model (PLAXIS 3D)	Validate analytical predictions using FEA model with same assumptions (such as full overburden load, elastic foundation). Compare moment profiles and identify limitations of analytical model if any.
Section 4.3 Soil Structure interaction models	Analytical vs. Soil-Structure Interaction FEA Model	Compare the moment distribution of beam on elastic foundation models with 3D FEA models.
	FEA Model – Variation of Tunnel Heading Height (7 m vs. 8.5 m)	Assess the influence of tunnel heading height on bending moment predictions using FEA models.

Initially, the analytical methods were evaluated against each other for identical tunnel advance length and uniform full overburden load, for two ground types, relatively stiff Class V sandstone and a weaker Class V shale. A tube spacing of 350mm centre to centre was assumed for all calculations. The rock mass properties adopted are detailed in subsequent sections. The full overburden load was directly applied, thereby eliminating the influence of rock mass properties typically required for calculating silo pressure. Rock mass properties also do not affect the boundary conditions for simplified beam models with fixed or simply supported boundaries. However, they are critical in beam-on-elastic-foundation models, where deformation depends on foundation stiffness.

Following this, a validation model was developed using PLAXIS 3D FEA software, as shown in Figure 2a. This model replicates the assumptions of Wang & Jia (2008) and allows for direct comparison of results between the analytical beam on elastic foundation model and the finite element model under identical conditions with full overburden load imposed.

The impact of varying the applied loads was explored for both analytical methods, and a simplified ground-structure finite element model, simulating a single tunnel advance as shown in Figure 2b was developed. The following assumptions were made for the finite element model:

– A stiff shotcrete liner as shown in blue in Figure 2b was assumed to support the canopy tubes over the excavated region. Similar to the analytical models, the edge of the liner terminates 1.5 times the unsupported length away from the face of the tunnel (i.e. behind the actual leading edge of the installed support).
– Model simulations are based on Mohr-Coulomb parameters. The equivalent Mohr-Coulomb parameters are derived from the Hoek-Brown criterion, calibrated for the low confining stresses as shown in Table 2.
– The adopted cohesion values, particularly for Class V Shale, were low, which contributed to face instability in preliminary analyses. To achieve model convergence, cohesion values were increased ahead of the tunnel face to induce a more elastic response, while maintaining low stiffness. This approach enabled controlled deformation towards the tunnel face without

triggering model failure. Detailed analyses involving support systems such as dowel bars or shotcrete for tunnel face stability are not included in this study.

– Embedded beam elements in PLAXIS 3D are utilised to model individual canopy tubes as circular hollow tubes.

The outcomes of these comparisons, along with the key differences identified, are discussed in detail in the following sections.

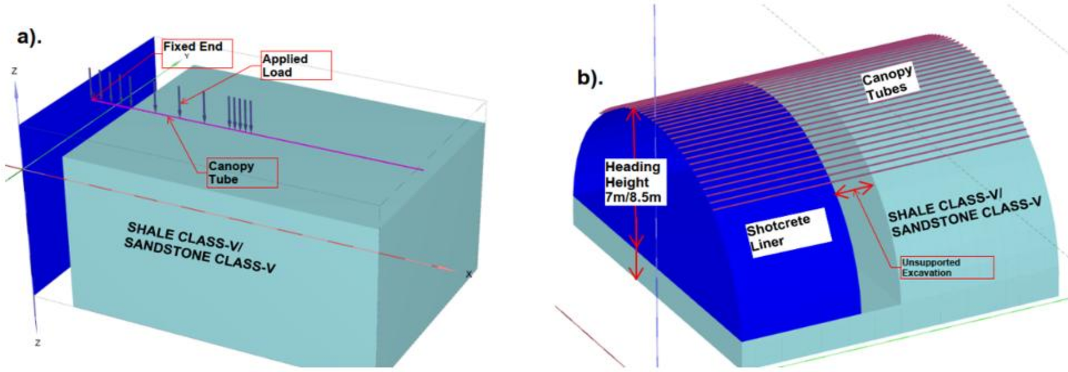

Figure 2. a). Validation model developed in FEA software b). Soil structure interaction model in FEA software

3.1 *Rock mass and pipe reinforcement properties*

Sydney Sandstone parameters are based on Bertuzzi (2014) classifications commonly used in local tunnel design. Class V Sandstone and Class V Shale were chosen to represent poor ground where canopy tubes are typically applied. Rock mass properties (Table 2) use Mohr–Coulomb parameters for both analytical load calculations and FEA. The canopy tube model is elastic, 114 mm OD, 6.3 mm wall, 15 m length, 210 GPa modulus, axial skin resistance 5000, set high to avoid local ground failure in unsupported spans.

Table 2. Rock parameters adopted in 3D FEM analysis

Geotechnical Parameters			Ashfield Shale Class V(SH-V)	Sandstone Class V(SS-V)
Intact Rock Parameters	Uniaxial compressive strength (MPa)		1	5
	Poisson's Ratio		0.3	0.3
	Material constant (m_i)		8	12
	Unit Weight (kN/m^3)		24	
Continuum parameters	GSI		20	35
	Rock Mass Modulus E_{mass} (MPa)		15	100
	Mohr-Coulomb$^{(*)}$	c' (kPa)	9	26
		ϕ'(°)	30	38

*Confining stress range of 0.05 to 0.25MPa assumed based on tunnel overburden depth.

4 ASSESSMENT RESULTS

The following section presents the results obtained from both analytical and FEA simulations. Trends in bending moments are compared with respect to variations in ground conditions, tunnel geometry, and applied loading scenarios.

4.1 *Comparison of analytical models*

Figure 3a presents bending moment distributions for a canopy tube under 10 m overburden using John & Mattle (2002) and Wang & Jia (2008). Both methods produce broadly comparable peak magnitudes (within ~10–15%) but differ in profile. The John & Mattle simplified beam model, independent of ground stiffness and based solely on the applied load, does not compute moments ahead of the tunnel face, giving identical results across ground conditions. In contrast, the beam-on-elastic-foundation model provides a profile extending ahead of the face. In weaker ground, such as Class V Shale, reduced stiffness increases the effective span, producing higher peak moments than the simplified beam approach. In both methods, maximum moments occur at rigid boundaries—the face for the simplified beam model and the leading edge of the initial support for the elastic foundation model.

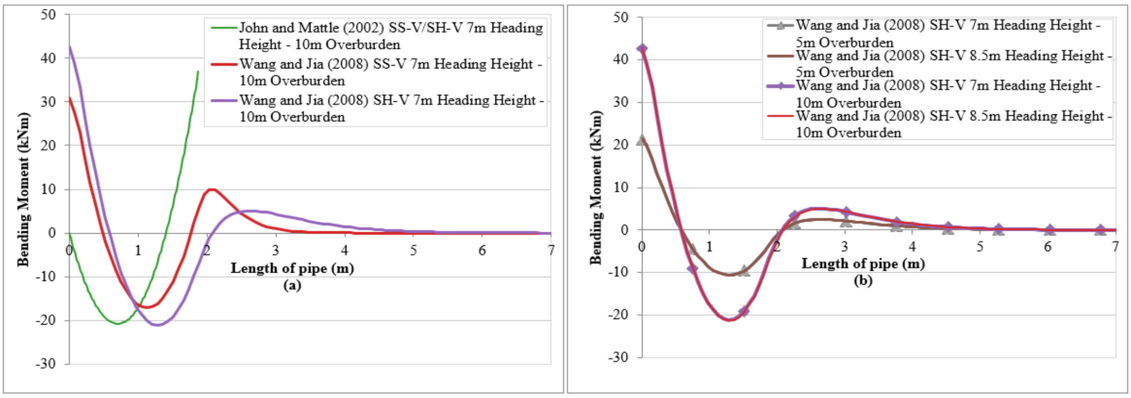

Figure 3. a). Predicted bending moment profile along the canopy tube based on John & Mattle(2002 and Wang & Jia (2008) b). Bending moment profile for tunnel heading heights 7m and 8.5m (plots are overlapping on top of each other) for various overburdens.

Wang & Jia (2008) suggests that the failure zone ahead of the tunnel is dependent on the tunnel heading height ('d' as shown in Figure 1b. To assess the influence of heading height two tunnel heading heights were analysed, 7m and 8.5m. The results (Figure 3b) show that increasing the heading height had no impact on the predicted bending moments.

The comparison presented in Figure 3b assumes full overburden loading. In Figure 4b, ground loads for Class V Shale were recalculated for tunnel overburdens of 5m, 10m, and 15m using Terzaghi's silo theory Wang & Jia (2008), Wang et al. (2009), and John & Mattle (2002). The results show a substantial reduction in maximum bending moments of about 50% for shallow tunnels. This corresponds directly to a reduction in vertical ground pressure, which decreases by approximately 50% for shallow tunnels and up to 75% for deeper tunnels. However, for shallow tunnels, the typical design practice is to consider full overburden pressure to account for ground variability. In line with the premise of the silo-theory, there is no significant increase in load beyond a certain depth.

4.2 *Idealised FEA model*

The analytical model (Figure 1b) was compared with an idealised FEA beam-on-elastic-foundation model (Figure 2a). John & Mattle (2002) was excluded, as its propped cantilever behaviour and profiles are already well understood. Figure 4a compares bending moment profiles from FEA and elastic foundation models for Class V Shale and Sandstone under 5 m and 10 m overburden. Agreement is generally good, but discrepancies appear near the fixed end. These are attributed to the central difference method used in the analytical model, which estimates moments from deflections using data points beyond the beam's physical limits, introducing larger boundary errors.

4.3 *Soil structure interaction model*

In this section, the results from a FEA soil structure interaction model using PLAXIS3D 2024.2 (Figure 2b) are presented. The model explicitly models individual canopy tube elements supported on the leading edge of the stiff tunnel liner with a free end embedded ahead of the tunnel face. To reduce computational demands, the staged excavation process was not modelled; instead, overburden pressures were applied in a single load step. Figure 5a presents the results of this analysis for the weaker ground condition (Class V Shale).

FEA and analytical results (using Terzaghi's silo load) yield similar peak bending magnitudes but differ in distribution. In weak ground, FEA shows peaks near the tunnel face, reflecting face extrusion effects not captured in the analytical models, which predict peaks at the fixed end. In competent sandstone, FEA results match Wang & Jia (2008), with peaks near the lining. Increasing heading height raises structural demand, an effect modelled in Wang & Jia's deeper failure plane but absent in the analytical results.

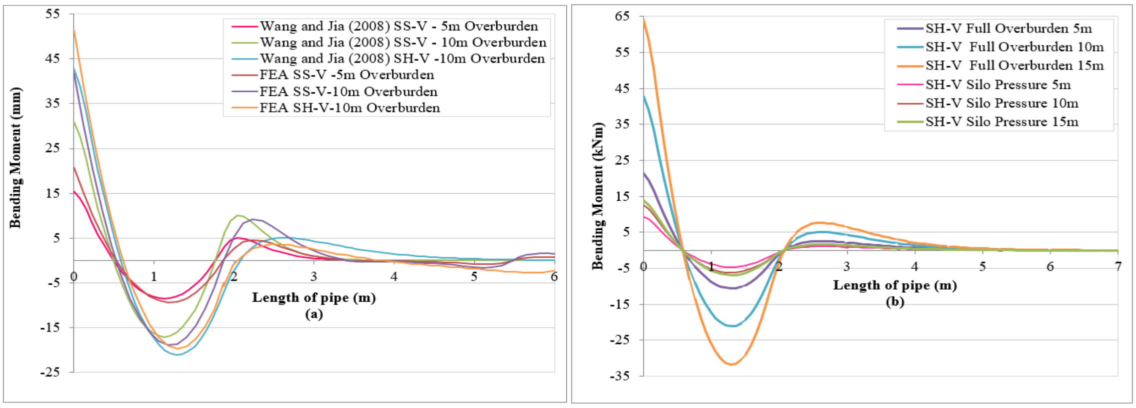

Figure 4. a). Idealised PLAXIS3D model vs. elastic foundation model predictions (Class V Shale and Sandstone) b). Moment profile generated using Wang & Jia (2008) for full overburden pressure vs silo theory-based pressures

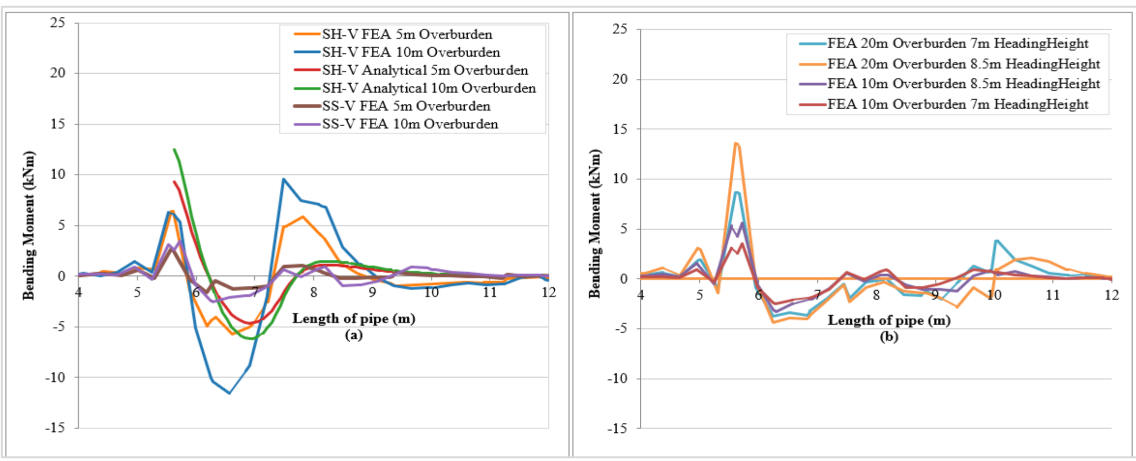

Figure 5. a) Results from the idealised FEA model compared to those of the elastic beam foundation model with Terzaghi's silo pressure load for Class V Shale, b). FEA model for Class V Sandstone comparing the results of 7 & 8.5m Heading height

5 CONCLUSION

Two analytical methods are evaluated alongside an equivalent numerical model representative of typical Sydney geology. The results of the analyses and review suggest the following.

- The applied load is a critical factor influencing the structural actions of canopy tubes; however, it may be challenging to estimate it accurately. Modelling suggests that Terzaghi's silo theory provides a reasonable approximation.
- Both analytical methods were effective at predicting peak bending moments reasonably well, though they differ in assumed boundary conditions and load transfer mechanisms. The method proposed by Wang & Jia (2008) showed closer agreement with FEA results, for weaker ground conditions ahead of the tunnel face.
- Pre-support systems implicitly rely on the face as a means of supporting the ground loads associated with potentially unstable ground ahead of the installed supports. Hence, maintenance of face stability is crucial.
- The assumption of fixed boundaries in both analytical approaches leads to a conservative estimate of maximum moment. It is acknowledged, and the FEA results show, that in effect each abutment of the pre-support system is in fact soft to some degree, with flexure of the lining (and also importantly early age strengths) and deformation of the ground at the face leading to lesser moments than predicted by the analytical methods.
- An increase in tunnel heading height may lead to higher face instability, face deformation leading to higher structural actions in canopy tube, this effect is not directly captured by analytical methods.

In summary, the comparison between analytical methods and FEA demonstrates that analytical approaches can offer a robust estimation of bending moments. Adequate support of the tubes via the initial lining and tunnel face is vital, often the face stability being the critical factor. Designs must adhere to applicable national standards and consider key elements such as tube connections, overlap with subsequent arrays, and ensuring that structural response remains within the elastic range to achieve a safe and robust solution

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